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| **FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY** | **SUBJECT:** DISCRETE STRUCTURE & APPLICATIONS | | |
| **CODE:** BCT2083 | **TOPIC:**  Chapter 3 Graphs | |
| **TUTORIAL 3** | | **DURATION:** 3 weeks (week 9-11) |

**Week 9**

1. Draw graphs models, stating the type of graph used, to represent airline routes where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flight from Washington to Newark and one flight from Washington to Miami, with

1. an edge between vertices representing cities that have a flight between them (in either direction)
2. an edge between vertices representing cities for each flight that operates between them (in either direction)
3. an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.
4. an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.
5. an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

2. Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of the graph

*d*

*c*

*b*

*a*

*d*

*c*

*b*

*a*

(a) (b)

*d*

*c*

*b*

*e*

*a*

*f*

*e*

*d*

*c*

*b*

*a*

(c) (d)

3. For each in question 2 that is not simple, find a set of edges to remove to make it simple.

4. Find the number of vertices, the number of edges and the degree of each vertex in the given undirected graph. Identify all the pendant and isolated vertices.

*c*

*b*

*a*

(a)

*f*

*e*

*d*

*e*

*d*

*c*

*b*

*a*

(b)

(c)

*i*

*h*

*g*

*e*

*d*

*c*

*b*

*a*

*f*

5. Find the in-degree and out-degree of each vertex for the given directed graph.

*d*

*c*

*b*

*a*

(a)

(b)

*d*

*c*

*b*

*a*

6. Determine whether the graph is bipartite.

*e*

*c*

*d*

*b*

*a*

(a)

*f*

*e*

*d*

*b*

*c*

*a*

(b)

*f*

*e*

*d*

*b*

*c*

*a*

(c)

7. Suppose that there are five young women and six young men on an island. Each woman is willing to marry some of the men on the island and each man is willing to marry any woman who is willing to marry him. Suppose that Ana is willing to marry Jason, Larry, and Matt; Barbara is willing to marry Kevin and Larry; Carol is willing to marry Jason, Nick and Oscar; Diane is willing to marry Jason, Larry , Nick and Oscar; and Elizabeth is willing to marry Jason and Matt.

(a) Graph the possible marriages on the island using a bipartite graph.

(b) Find a matching of the young women and the young men on the island such that each young woman is matched with a young man whom she is willing to marry

8. Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same).

*d*

*b*

*c*

*a*

*g*

*f*

*e*

*e*

*d*

*b*

*c*

*a*

(a)

*b*

*c*

*d*

*a*

*b*

*c*

*d*

*a*

(c)

9. Use an adjacency list to represent he given graph.

*b*

*e*

*d*

*c*

*a*

(a)

(b)

*e*

*f*

*a*

*b*

*c*

*d*

*d*

*b*

*c*

*a*

(c)

10. Represent the graph in question (9) with an adjacency matrix.

11. Draw a graph represented by the given adjacency matrix

(a)  (b) 

(c)  (d) 

*a*

12. Represent graphs below by using incidence matrix

*e5*

*e4*

*e12*

*e7*

*e6*

*e8*

*e10*

*e2*

*e3*

*e9*

*e*

*f*

*a*

*b*

*c*

*g*

*e1*

*h*

*d*

*e13*

(a)

*e11*

*e5*

*e4*

*e12*

*e7*

*e6*

*e8*

*e10*

*e2*

*e9*

*e13*

*e1*

*e11*

*a*

*d*

*e*

*b*

*c*

(b)

*e3*

*e5*

*e4*

*e7*

*e6*

*e8*

*e2*

*e3*

*e1*

*f*

*c*

*d*

*e*

*b*

*a*

(c)

**Week 10**

13. Determine whether the given pair of graphs is isomorphic.

*v3*

*v5*

*v2*

*v4*

*v1*

*u3*

*u1*

*u5*

*u2*

*u4*

(a)

(b)

*v3*

*v5*

*v2*

*v4*

*v1*

*u5*

*u3*

*u1*

*u2*

*u4*

*d*

*b*

*u8*

*u5*

*u7*

*u3*

*u1*

*u6*

*u2*

*u4*

(c)

*v8*

*v7*

*v6*

*v3*

*v5*

*v2*

*v4*

*v1*

14. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

*e*

*b*

*c*

*d*

*a*

(a) (i) *a, e, b, c, b*

(ii) *a, e, a, d, b, c, a*

(iii) *e, b, a, d, b, e*

(iv) *c, b, d, a, e, c*

*b*

*e*

*d*

*a*

*c*

(b) (i) *a, b, e, c, b*

(ii) *a, d, a, d, a*

(iii) *a, d, b, e, a*

(iv) *a, b, e, c, b, d, a*

15. Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

*c*

*d*

*b*

*e*

*a*

*f*

*e*

*a*

*b*

*d*

*c*

1. (b)

*g*

*c*

*d*

*b*

*e*

*a*

*f*

*g*

*c*

*d*

*b*

*e*

*a*

*f*

1. (d)

16. Find the strongly components of each of these graphs

*c*

*b*

*a*

*e*

*d*

*f*

*c*

*d*

*b*

*e*

*a*

*f*

*h*

*g*

*i*

(a)

(b)

*c*

*b*

*a*

*g*

*f*

*h*

*e*

*d*

(c)

*a*

*b*

*h*

*e*

*d*

*c*

*i*

*g*

*f*

(d)

17. Find the number of paths of length *n* between two different vertices in *K4* if *n* is

(a) 2 (b) 3 (c) 4 (d) 5

18. Determine if the following graph are isomorphic.

*u6*

*u7*

*u5*

*u3*

*u1*

*u2*

*u4*

*u8*

*v5*

*v3*

*v2*

*v4*

*v1*

*v7*

*v6*

*v8*

*u3*

(a)

*u6*

*u7*

*u5*

*u1*

*u2*

*u4*

*u8*

*u3*

*v5*

*v3*

*v2*

*v4*

*v1*

*v7*

*v6*

*v8*

(b)

19. Find all the cut vertices of the given graph

*c*

*b*

*h*

*g*

*i*

*d*

*e*

*a*

*f*

*d*

*b*

*a*

*e*

*c*

*f*

(a) (b)

20. Find all the cut edges from the graphs in question (19)

21. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such path if one exists.

*e*

*d*

*f*

*a*

*c*

*b*

*i*

*e*

*g*

*f*

*h*

*a*

*b*

*d*

*c*

1. (b)

*o*

*m*

*n*

*l*

*k*

*j*

*e*

*d*

*c*

*b*

*a*

*h*

*i*

*g*

*f*

(c)

*c*

*d*

*b*

*a*

(d)

*c*

*d*

*b*

*a*

*e*

(e)

22. Determine whether the given graph has a Hamilton circuit. Construct such a circuit when one exists. If no Hamilton circuit exists, determine whether the graph has an Hamilton path and construct such path if one exists.

*c*

*d*

*b*

*a*

*f*

*e*

*f*

(a)

*d*

*b*

*a*

*c*

*e*

(b)

(c)

*j*

*m*

*q*

*i*

*l*

*p*

*c*

*d*

*k*

*h*

*g*

*n*

*e*

*b*

*a*

*f*

*o*

**Week 11**

23. By using Djikstra’s algorithm, find a route with the least total airfare that visits each of the cities in this graph, where the weight on an edge is the least price (RM) available for a flight between the two cities.

1. from San Francisco to New York

San Francisco

Detroit

New York

Los Angeles

Denver

69

179

359

349

209

229

329

189

279

379

1. from Seattle to New York

Seattle

Phoenix

New Orleans

New York

Boston

319

379

429

389

119

409

229

309

109

239

24. Determine whether the given graph is planar. If so, draw it so that no edges cross.

*c*

*b*

*d*

*a*

*e*

*d*

*a*

*c*

*e*

*b*

1. (b)

*h*

*g*

*c*

*d*

*b*

*a*

*f*

*e*

*c*

*d*

*b*

*a*

*f*

*e*

1. (d)

25. Use Kuratowski’s Theorem to determine whether the given graph is planar.

*c*

*d*

*g*

*b*

*a*

*f*

*e*

*h*

*c*

*d*

*g*

*b*

*a*

*f*

*e*

(a) (b)

26. Find the number of colours needed to colour the map so that no two adjacent regions have the same colour.

A

B

C

D

E

(a)

(b)

A

B

C

D

E

F

*d*

*b*

*a*

*c*

(c)

**ANSWERS CHAPTER 3: Graph**

1. Answer not unique

2. a) simple graph b) pseudograph c&d) directed multigraph

3. b) ({*a*}, {*b*}, {*d*}, {*a,b*}, {*c,d*}) c) ({*a,b*}, {*b,c*}, {*c,d*}, {*c,d*}, {*a*}, {*e*}, {*a,e*}) d) ({*a,b*}, {*b,c*}, {*e,d*}, {*f*})

4. pendant: a) *c* b) 0 c) 0 isolated: a) *d* b) 0 c) *d, i*

5. a) in-degree: 2,1,2,1 out-degree: 0,2,1,3 b) in-degree: 2,3,2,1 out-degree: 2,4,1,1

6. a) yes b) no c) yes

7. b) Ana&Matt, Barbara&Larry, Carol&Oscar, Dianne&Nick, Elizabeth&Jason

13. a) yes b) no c) no

14. a) (i) path, not simple, not circuit, 4 (ii) not path (iii) not path (iv) path, simple circuit, 5

b) (i) path, simple, not circuit, 4 (ii) path ,not simple, circuit, 4 (iii) not path (iv) not path

15. a) weakly b) neither c) weakly d) neither

16. a) {*a,b,h,i*}, {*c,d,f,g*}, {*e*} b) {*a,b,f*}, {*c,d,e*} c) {*a,b,c,d,e,h*}, {*f*}, {*g*} d) {*a,b,d,f,g,h,i*}, {*c*}, {*e*}

17. a) 2 b) 7 c) 20 d) 61

18. a) yes b) not

19. a) *c* b) *b,c,e,i*

20. a) none b) {*a,b*},{*b,c*}, {*c,d*}, {*c,e*}, {*e,i*}, {*i,h*}

21. a) euler circuit b) euler path c) euler circuit d) euler path e) euler circuit

22. a) Hamilton path b) Hamilton circuit c) neither

24. a) yes b) no c) yes d) no

25 a) yes b) no

26. a) 4 b) 4 c) 3